

Block warm-up (3/18)

$$18. \frac{\sec^2 \beta - 1}{\tan \beta} \cdot \frac{\cot \beta}{\csc^2 \beta - 1} = \frac{\tan^2 \beta}{\tan \beta} \cdot \frac{\cot \beta}{\cot^2 \beta} = \tan \beta \cdot \frac{1}{\cot \beta} = \tan \beta \cdot \tan \beta = \boxed{\tan^2 \beta}$$

$$20. \frac{1 + \tan^2 x}{1 + \cot^2 x} + \frac{1 - \cos^2 x}{1 - \sin^2 x} = \frac{\sec^2 x}{\csc^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{\frac{1}{\cos^2 x} \cdot \sin^2 x}{\frac{1}{\sin^2 x} \cdot \cos^2 x} + \tan^2 x = \frac{\sin^2 x}{\cos^2 x} + \tan^2 x = \tan^2 x + \tan^2 x = \boxed{2 \tan^2 x}$$

$$22. \frac{\tan x}{\sec x} + \frac{\csc x}{\sec x} \cdot \frac{1 + \sec x}{1 + \sec x} = \frac{\sec x \tan x + \csc x + \csc x \sec x}{\sec x (1 + \sec x)} = \frac{\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} + \frac{1}{\sin x} + \frac{1}{\sin x} \cdot \frac{1}{\cos x}}{\frac{1}{\cos x} (1 + \frac{1}{\cos x})}$$

$$= \left(\frac{\frac{\sin x}{\cos^2 x} + \frac{1}{\sin x} + \frac{1}{\sin x \cos x}}{\left(\frac{1}{\cos x} + \frac{1}{\cos^2 x} \right)} \right) \cdot \frac{\cos^2 x \sin x}{\cos^2 x \sin x} = \frac{\sin^2 x + \cos^2 x + \cos x}{\cos x \sin x + \sin x} = \frac{1 + \cos x}{\sin x (\cos x + 1)}$$

$$= \frac{1}{\sin x} = \boxed{\csc x}$$

$$\text{OR } \frac{\left(\frac{\sin x}{\cos x} \right) \cdot \cos x}{\left(1 + \frac{1}{\cos x} \right) \cdot \cos x} + \frac{\frac{1}{\sin x} \cdot \cos x}{\frac{1}{\cos x} \cdot \cos x} = \frac{\sin x \cdot \sin x}{\sin x (\cos x + 1)} + \frac{\cos x \cdot (\cos x + 1)}{\sin x (\cos x + 1)} = \frac{\sin^2 x + \cos^2 x + \cos x}{\sin x (\cos x + 1)}$$

$$= \frac{1 + \cos x}{\sin x (\cos x + 1)} = \frac{1}{\sin x} = \csc x$$

$$24. \frac{1}{\csc^2 x + \csc x \cot x} = \frac{1}{\frac{1}{\sin^2 x} + \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}} = \frac{1}{\frac{1}{\sin x} + \frac{\cos x}{\sin^2 x}} = \frac{(1) \cdot \sin^2 x}{(1 + \cos x) \cdot \sin^2 x}$$

$$= \frac{\sin^2 x}{1 + \cos x} = \frac{1 - \cos^2 x}{1 + \cos x} = \frac{(1 + \cos x)(1 - \cos x)}{1 + \cos x} = \boxed{1 - \cos x}$$